

Binary Numbers

Appendix A

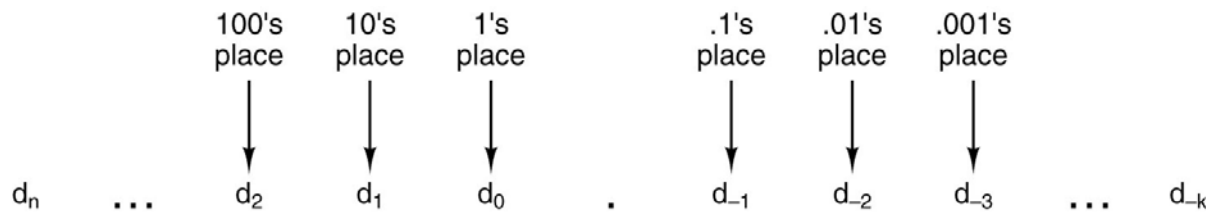
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Finite Precision Numbers

1. Numbers larger than 999
2. Negative numbers
3. Fractions
4. Irrational numbers
5. Complex numbers

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Radix Number Systems (1)



$$\text{Number} = \sum_{i=-k}^n d_i \times 10^i$$

The general form of a decimal number.

Radix Number Systems (2)

Binary	1	1	1	1	1	0	1	0	0	0	1
	1×2^{10}	$+ 1 \times 2^9$	$+ 1 \times 2^8$	$+ 1 \times 2^7$	$+ 1 \times 2^6$	$+ 0 \times 2^5$	$+ 1 \times 2^4$	$+ 0 \times 2^3$	$+ 0 \times 2^2$	$+ 0 \times 2^1$	$+ 1 \times 2^0$
	1024	+ 512	+ 256	+ 128	+ 64	+ 0	+ 16	+ 0	+ 0	+ 0	+ 1
Octal	3	7	2	1							
	3×8^3	$+ 7 \times 8^2$	$+ 2 \times 8^1$	$+ 1 \times 8^0$							
	1536	+ 448	+ 16	+ 1							
Decimal	2	0	0	1							
	2×10^3	$+ 0 \times 10^2$	$+ 0 \times 10^1$	$+ 1 \times 10^0$							
	2000	+ 0	+ 0	+ 1							
Hexadecimal	7	D	1								.
	7×16^2	$+ 13 \times 16^1$	$+ 1 \times 16^0$								
	1792	+ 208	+ 1								

The number 2001 in binary, octal, and hexadecimal.

Conversion from One Radix to Another(1)

Decimal numbers and their binary, octal, and hexadecimal equivalents.

Decimal	Binary	Octal	Hex
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E

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Conversion from One Radix to Another(2)

Decimal numbers and their binary, octal, and hexadecimal equivalents.

15	1111	17	F
16	10000	20	10
20	10100	24	14
30	11110	36	1E
40	101000	50	28
50	110010	62	32
60	111100	74	3C
70	1000110	106	46
80	1010000	120	50
90	1011010	132	5A
100	11001000	144	64
1000	1111101000	1750	3E8
2989	101110101101	5655	BAD

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Conversion from One Radix to Another(3)

Example 1

Hexadecimal

Binary

Octal

1	9	4	8	.	B	6										
0001		1001		0100		1000		.	1011		0110		00			
1		4		5		1		0		.	5		5		4	

Examples of octal-to-binary and hexadecimal-to-binary conversion.

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Conversion from One Radix to Another(4)

Example 2

Hexadecimal

Binary

Octal

7	B	A	3	.	B	C	4											
0111		1011		1010		0011		.	1011		1100		0100					
7		5		6		4		3		.	5		7		0		4	

Examples of octal-to-binary and hexadecimal-to-binary conversion.

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Conversion from One Radix to Another(5)

Conversion of the decimal number 1492 to binary by successive halving, starting at the top and working downward. For example, 93 divided by 2 yields a quotient of 46 and a remainder of 1, written on the line below

Quotients	Remainders
1492	
746	0
373	0
186	1
93	0
46	1
23	0
11	1
5	1
2	1
1	0
0	1

1 0 1 1 1 0 1 0 1 0 0 = 1492₁₀

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Conversion from One Radix to Another(6)

1	0	1	1	1	0	1	1	0	1	1	1
											1
											→ 1 + 2 × 1499 = 2999
											→ 1 + 2 × 749 = 1499
											→ 1 + 2 × 374 = 749
											→ 0 + 2 × 187 = 374
											→ 1 + 2 × 93 = 187
											→ 1 + 2 × 46 = 93
											→ 0 + 2 × 23 = 46
											→ 1 + 2 × 11 = 23
											→ 1 + 2 × 5 = 11
											→ 1 + 2 × 2 = 5
											→ 0 + 2 × 1 = 2
											→ 1 + 2 × 0 = 1

← Result

← Start here

Conversion of the binary number 101110110111 to decimal by successive doubling, starting at the bottom. Each line is formed by doubling the one below it and adding the corresponding bit. For example, 749 is twice 374 plus the 1 bit on the same line as 749.

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Representing Negative Numbers

- Signed magnitude
- One's complement
- Two's complement
- Excess $2^m - 1$

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Negative Binary Numbers (1)

N decimal	N binary	-N signed mag.	-N 1's compl.	-N 2's compl.	-N excess 128
1	00000001	10000001	11111110	11111111	01111111
2	00000010	10000010	11111101	11111110	01111110
3	00000011	10000011	11111100	11111101	01111101
4	00000100	10000100	11111011	11111100	01111100
5	00000101	10000101	11111010	11111011	01111011
6	00000110	10000110	11111001	11111010	01111010
7	00000111	10000111	11111000	11111001	01111001
8	00001000	10001000	11110111	11111000	01111000
9	00001001	10001001	11110110	11110111	01110111
10	00001010	10001010	11110101	11110110	01110110

Negative 8-bit numbers in four systems.

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Negative Binary Numbers (2)

N decimal	N binary	-N signed mag.	-N 1's compl.	-N 2's compl.	-N excess 128
20	00010100	10010100	11101011	11101100	01101100
30	00011110	10011110	11100001	11100010	01100010
40	00101000	10101000	11010111	11011000	01011000
50	00110010	10110010	11001101	11001110	01001110
60	00111100	10111100	11000011	11000100	01000100
70	01000110	11000110	10111001	10111010	00111010
80	01010000	11010000	10101111	10110000	00110000
90	01011010	11011010	10100101	10100110	00100110
100	01100100	11100100	10011011	10011100	00011100
127	01111111	11111111	10000000	10000001	00000001
128	Nonexistent	Nonexistent	Nonexistent	10000000	00000000

Negative 8-bit numbers in four systems.

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Binary Arithmetic (1)

Addend	0	0	1	1
Augend	<u>+0</u>	<u>+1</u>	<u>+0</u>	<u>+1</u>
Sum	0	1	1	0
Carry	0	0	0	1

The addition table in binary.

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Binary Arithmetic (2)

<u>Decimal</u>	<u>1's complement</u>	<u>2's complement</u>
$\begin{array}{r} 10 \\ + (-3) \\ \hline \end{array}$	$\begin{array}{r} 00001010 \\ 11111100 \\ \hline \end{array}$	$\begin{array}{r} 00001010 \\ 11111101 \\ \hline \end{array}$
$+7$	$\begin{array}{r} 1 \ 00000110 \\ \quad \swarrow \\ \quad \text{carry } 1 \\ \hline 00000111 \end{array}$	$\begin{array}{r} 1 \ 00000111 \\ \quad \downarrow \\ \quad \text{discarded} \end{array}$

Addition in one's complement and two's complement.